2

A young couple decide to build a house but the project is delayed due to ongoing poor weather and contractor issues. Each month, the couple make an estimate of the total cost of the house. The table shows the estimated cost, £y thousand, of the project t months after the project was started.

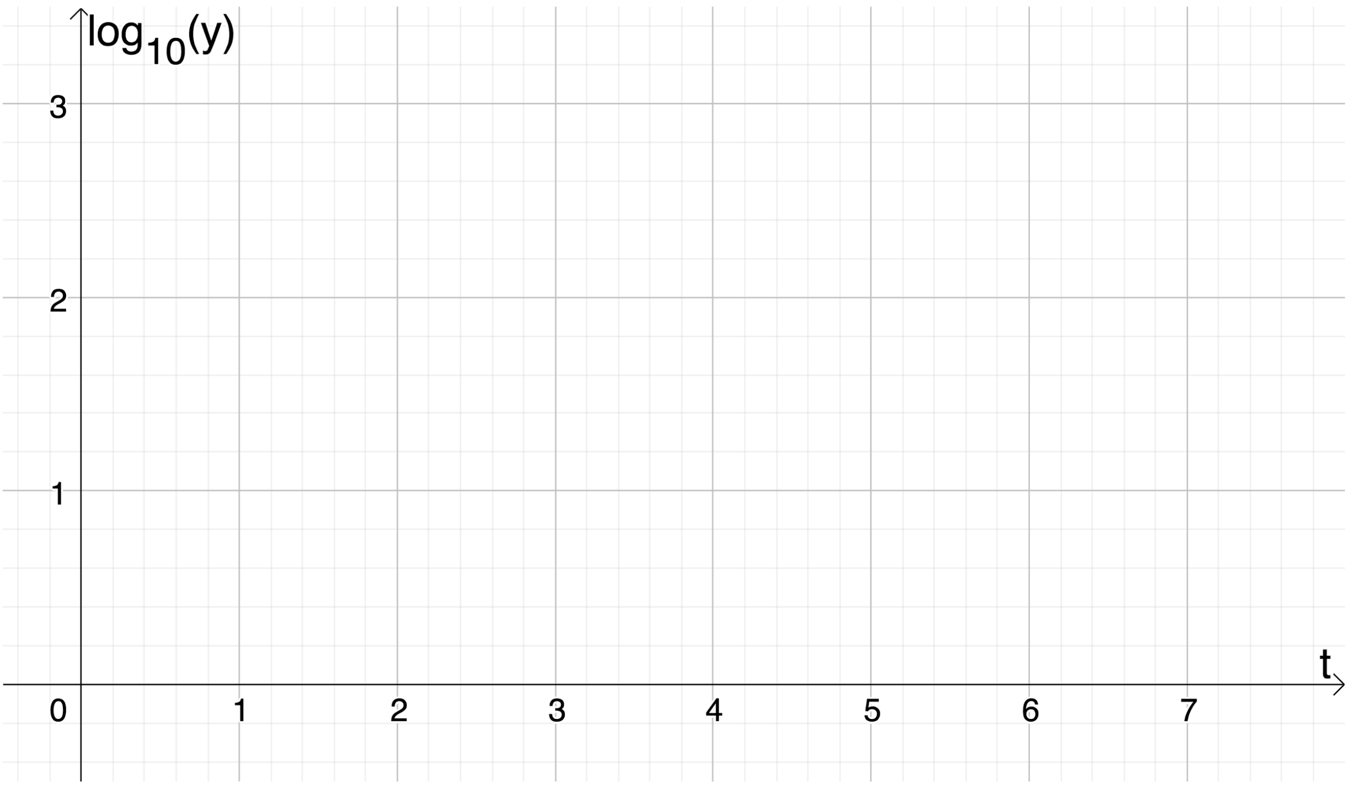
Months after the project was started, t	1	2	3	4	5	6
Cost, £y thousand	280	289	332	371	408	458

The relationship between y and t is modelled by $y = ab^t$, where a and b are constants.

a. Show that $y = ab^t$ may be written as
$$\log_{10} y = \log_{10} a + t \log_{10} b$$

b. Complete the table of values below and plot $\log_{10} y$ against t, drawing by eye a line of best fit.

t	1	2	3	4	5	6
y	280	289	332	371	408	458
$\log_{10} y$						



c. Use your graph to find the equation for y in terms of t.

d. Find the value of t given by this model when the estimated cost is £600 thousand. Give your answer rounded to 1 decimal place.

3

The table shows the estimated population of India during the period 1950 to 2000.

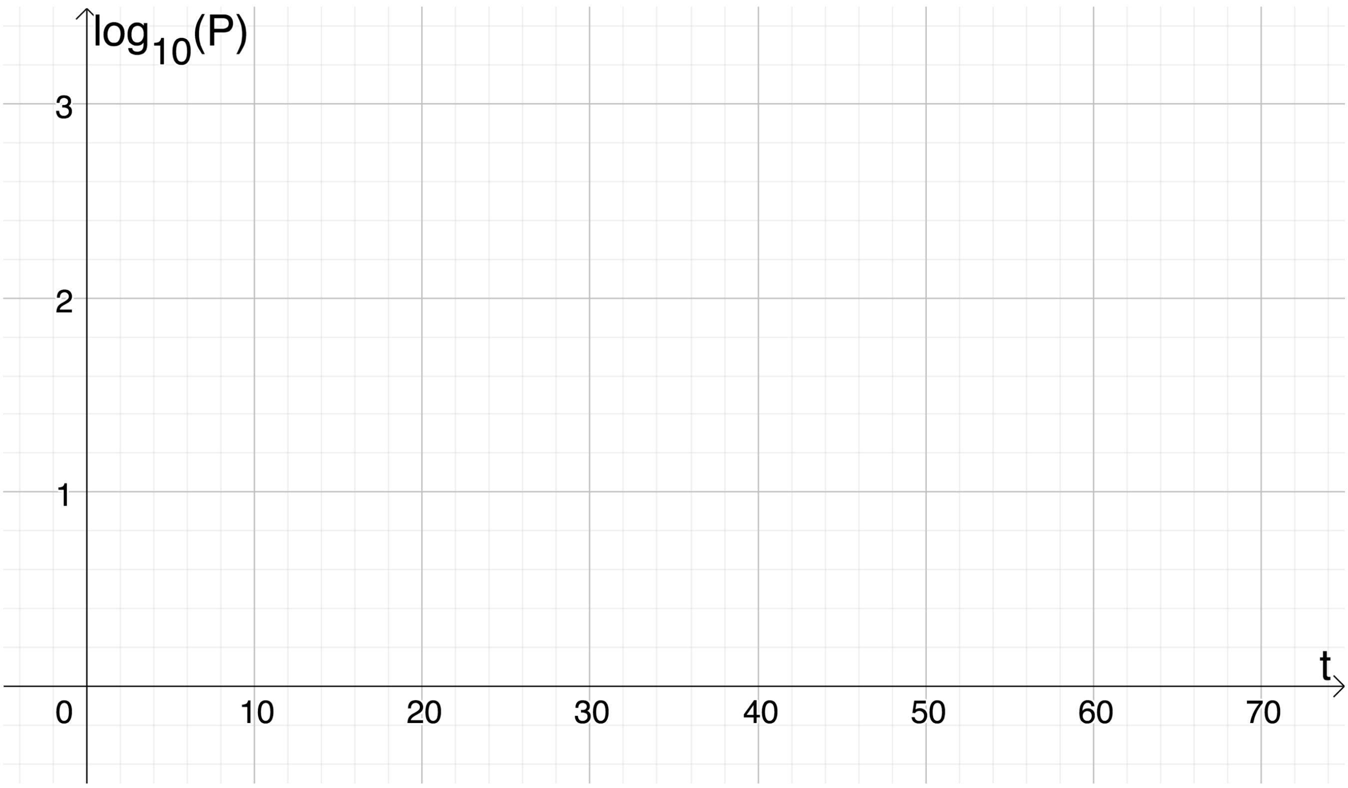
Year	1950	1960	1970	1980	1990	2000
Population (P millions)	376	451	555	699	873	1057

The population is modelled by $P = ab^t$, where the population P is in millions, t is the number of years after 1940, and a and b are constants.

a. Show that, using this model, the graph of $\log_{10} P$ against t is a straight line of gradient b. State the intercept of this line on the vertical axis.

b. Complete the table of values below and plot $\log_{10} P$ against t, drawing by eye a line of best fit.

t	10	20	30	40	50	60
P	376	451	555	699	873	1057
$\log_{10} P$						



c. Use your graph to find the equation for P in terms of t.

d. Use your results to estimate the population of India in 2030. Comment, with a reason, on the reliability of this estimate.

4

A virus affected a particular species of monkey so that its population severely declined between 1990 and 2010. Estimated population figures are shown in the table below.

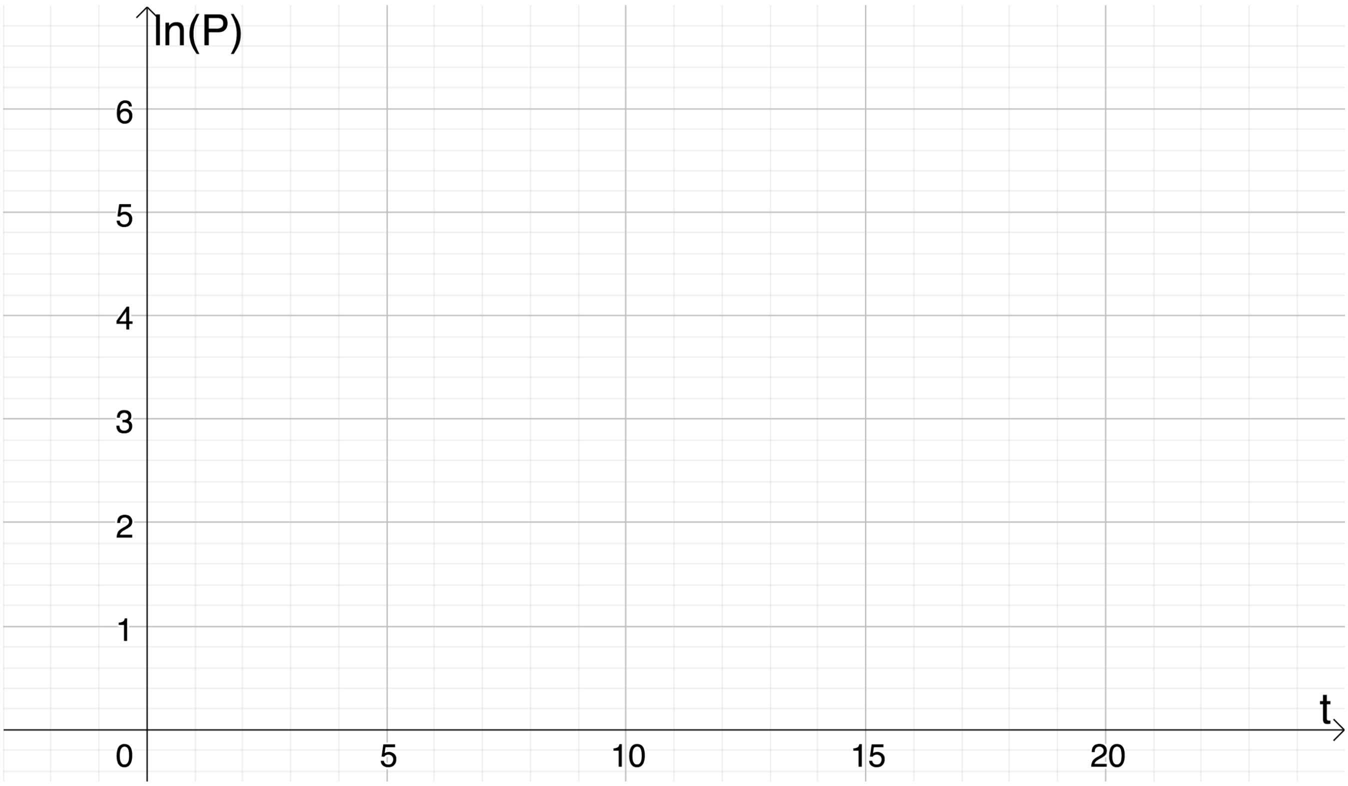
Year	1990	1994	1998	2002	2006	2010
Population (P thousands)	259	190	111	79	47	31

The population is modelled by $P = ab^t$, where the population P is in thousands, t is the number of years after 1990, and a and b are constants.

a. Show that, using this model, the graph of $\ln P$ against t is a straight line of gradient b. State the intercept of this line on the vertical axis.

b. Complete the table of values below and plot $\ln P$ against t, drawing by eye a line of best fit.

t	0					
P	259	190	111	79	47	31
$\ln P$						



c. Use your graph to find the equation for P in terms of t.

d. Use your results to estimate the year in which the population will drop to 10,000.

6

The table shows the estimated population of the world, in millions, from 1950 to 2015.

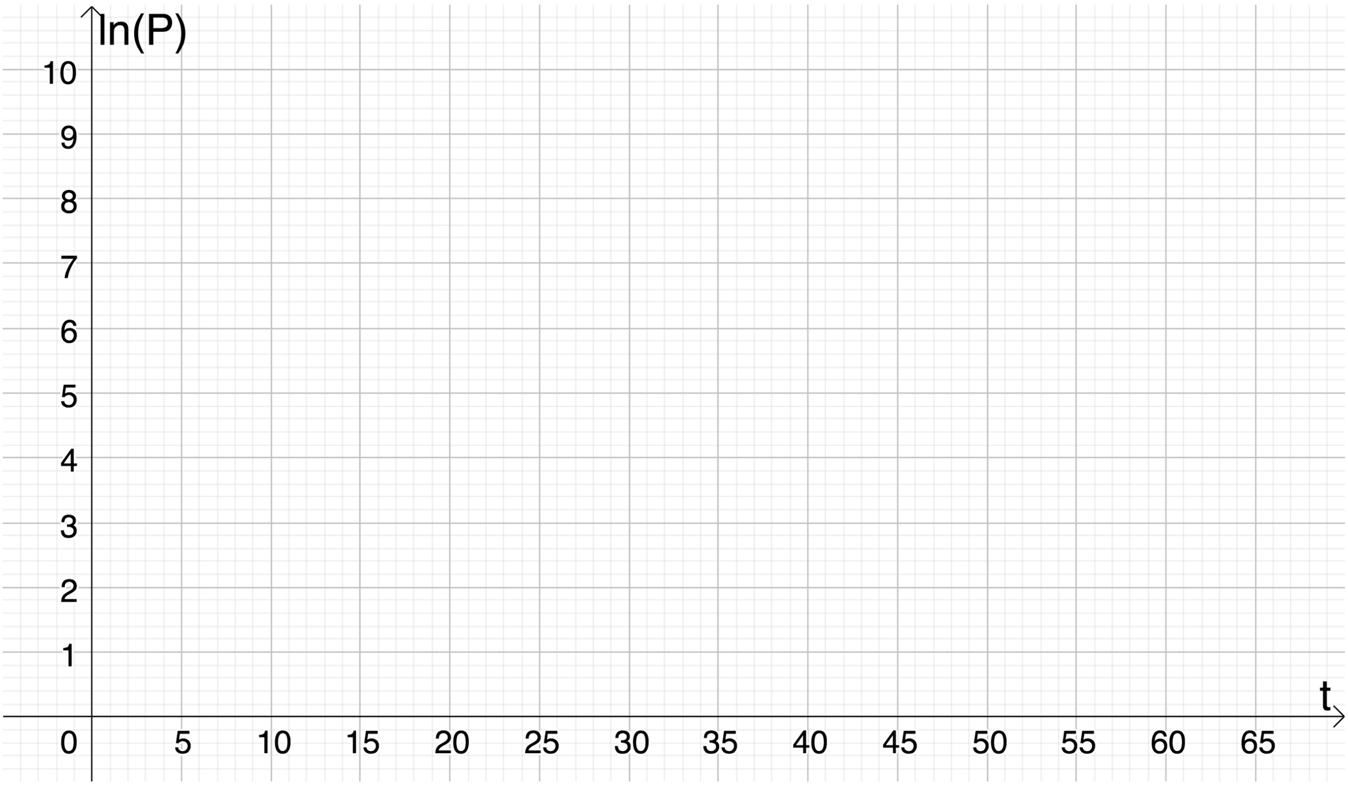
Year	1950	1960	1980	2000	2010	2015
Population (P millions)	2525	3018	4440	6127	6930	7359

The population is modelled by $P = ab^t$, where the population P is in millions, t is the number of years after 1950, and a and b are constants.

a. Show that, using this model, the graph of $\ln P$ against t is a straight line of gradient b. State the intercept of this line on the vertical axis.

b. Complete the table of values below and plot $\ln P$ against t, drawing by eye a line of best fit.

t	0	10	30	50	60	65
P	2525	3018	4440	6127	6930	7359
$\ln P$						



c. Use your graph to find the equation for P in terms of t.

d. The UN projects that the population of the world will be 9735 million by 2050. Use your results to estimate the population of the world in 2050, and find the percentage difference between this and the UN's projected figure.

7

The table below shows the number of new cases of measles reported in the United States between 1950 and 2000.

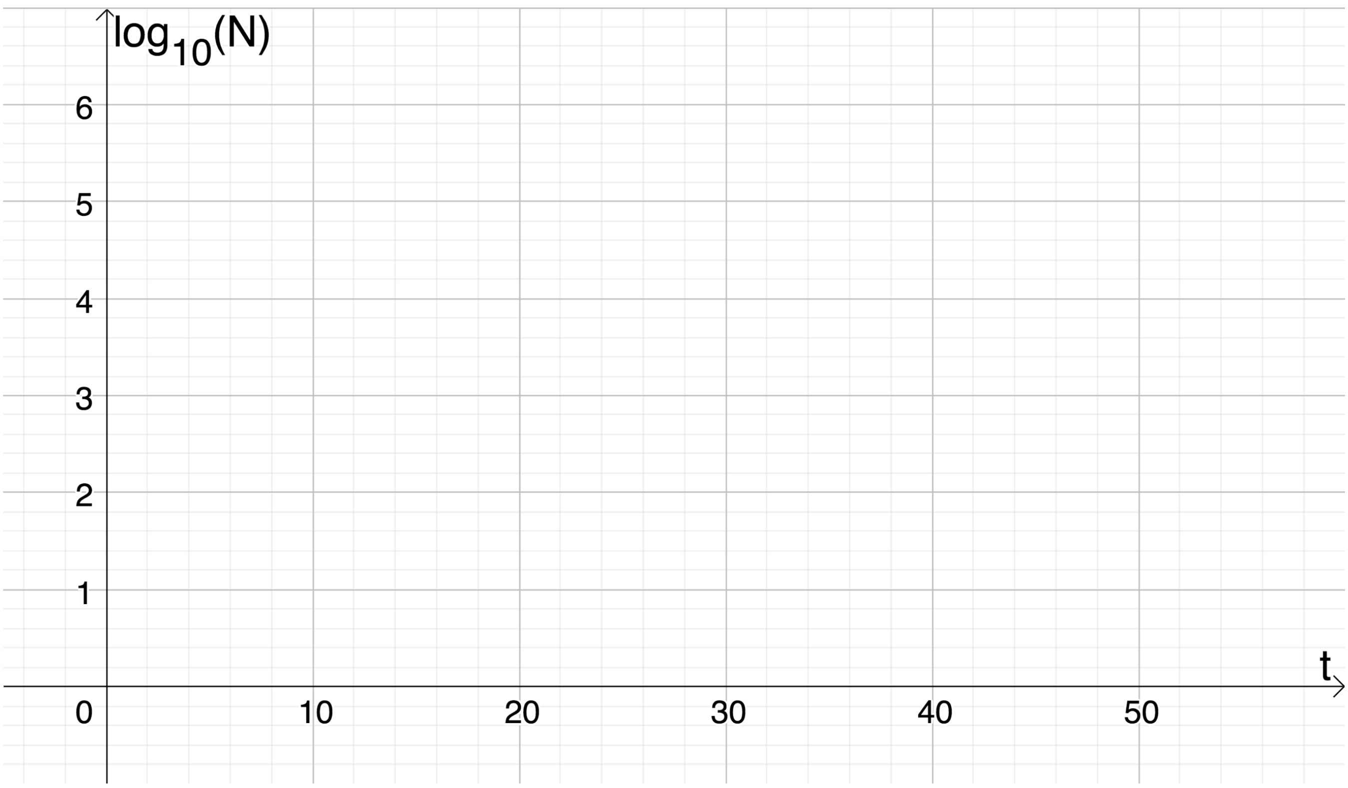
Year	1950	1960	1970	1980	1990	2000
Number (N)	319,124	441,703	47,351	13,506	27,786	86

The population is modelled by $N = ab^t$, where N is the population, t is the number of years after 1950, and a and b are constants.

a. Show that, using this model, the graph of $\log_{10} N$ against t is a straight line of gradient b. State the intercept of this line on the vertical axis.

b. Complete the table of values below and plot $\log_{10} N$ against t, drawing by eye a line of best fit.

t						
N	319,124	441,703	47,351	13,506	27,786	86
$\log_{10} N$						



c. Use your graph to find the equation for N in terms of t.

2

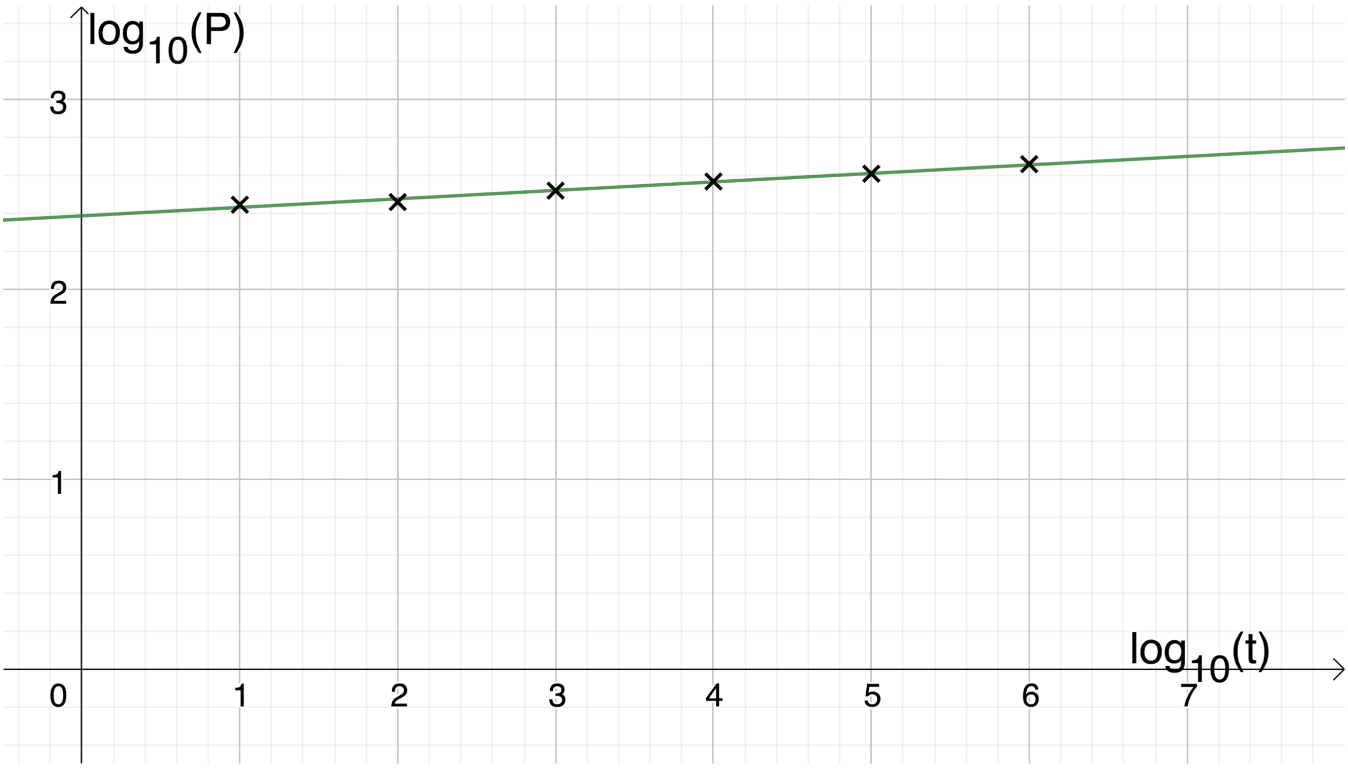
a. Show that $y = ab^t$ may be written as

$$\log_{10} y = \log_{10} a + t \log_{10} b$$

$$y = ab^t \Rightarrow \log_{10} y = \log_{10}(ab^t)$$
$$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} b^t$$
$$\Rightarrow \log_{10} y = \log_{10} a + t \log_{10} b$$

b. Complete the table of values below and plot $\log_{10} y$ against t, drawing by eye a line of best fit.

t	1	2	3	4	5	6
y	280	289	332	371	408	458
$\log_{10} y$	2.44716	2.4609	2.52114	2.56937	2.61066	2.66087



c. Use your graph to find the equation for y in terms of t.

Gradient ≈ 0.0447
Intercept ≈ 2.39

$$P = 245 \times 1.11^t$$

d. Find the value of t given by this model when the estimated cost is £600 thousand. Give your answer rounded to 1 decimal place.

$$600 = 245 \times 1.11^t$$
$$t = \log_{1.11}\left(\frac{600}{245}\right) = 8.6 \text{ months to 1dp}$$

3

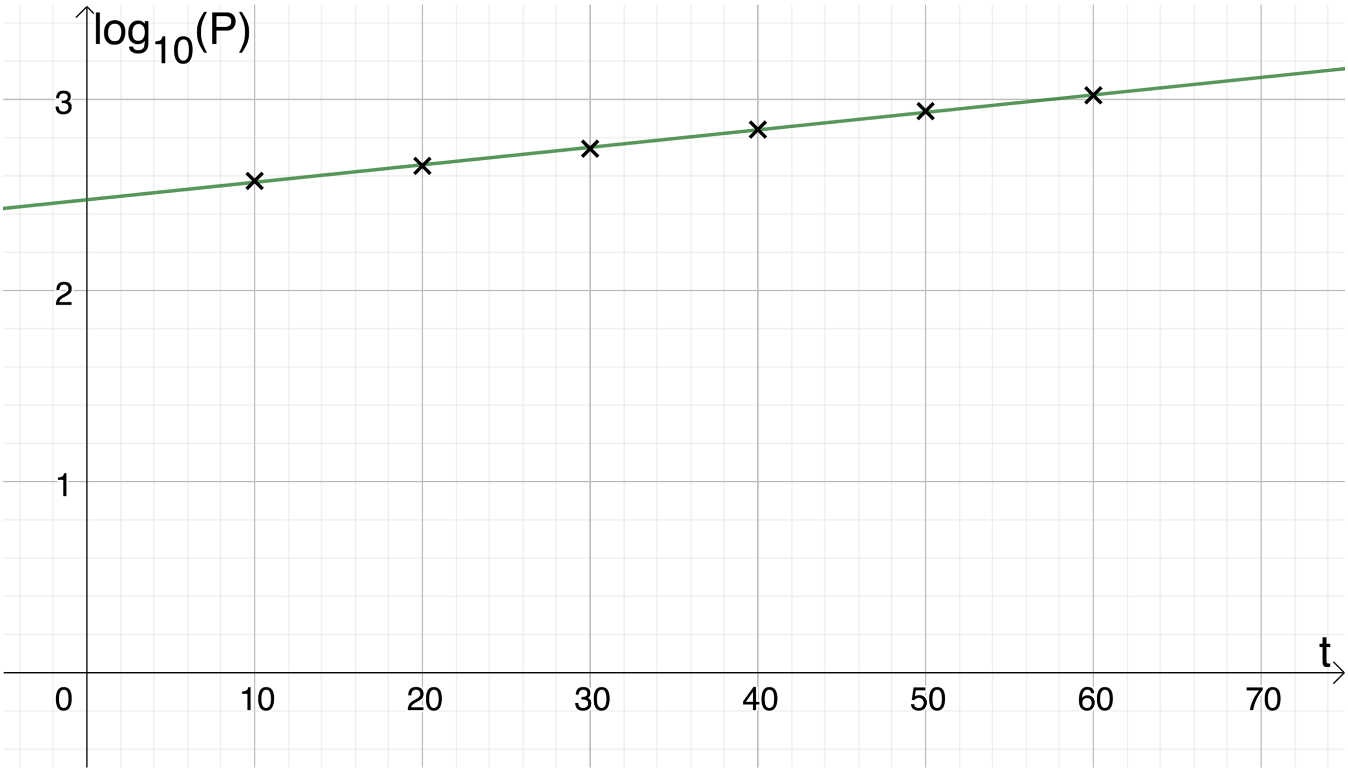
a. Show that, using this model, the graph of $\log_{10} P$ against t is a straight line of gradient b. State the intercept of this line on the vertical axis.

$$P = ab^t \Rightarrow \log_{10} P = \log_{10}(ab^t)$$
$$\Rightarrow \log_{10} P = \log_{10} a + \log_{10} b^t$$
$$\Rightarrow \log_{10} P = \log_{10} a + t \log_{10} b$$

Intercept on the vertical axis is $\log_{10} a$

b. Complete the table of values below and plot $\log_{10} P$ against t, drawing by eye a line of best fit.

t	10	20	30	40	50	60
P	376	451	555	699	873	1057
$\log_{10} P$	2.57519	2.65418	2.74429	2.84448	2.94101	3.02407



c. Use your graph to find the equation for P in terms of t.

$$P = 299.706 \times 1.021^t$$

d. Use your results to estimate the population of India in 2030. Comment, with a reason, on the reliability of this estimate.

$$P|_{t=90} \approx 1945 \text{ million}$$

It is not necessarily the case that the pattern will continue in the way that it has. The time is far ahead of the recorded data and so may not be reliable.

4

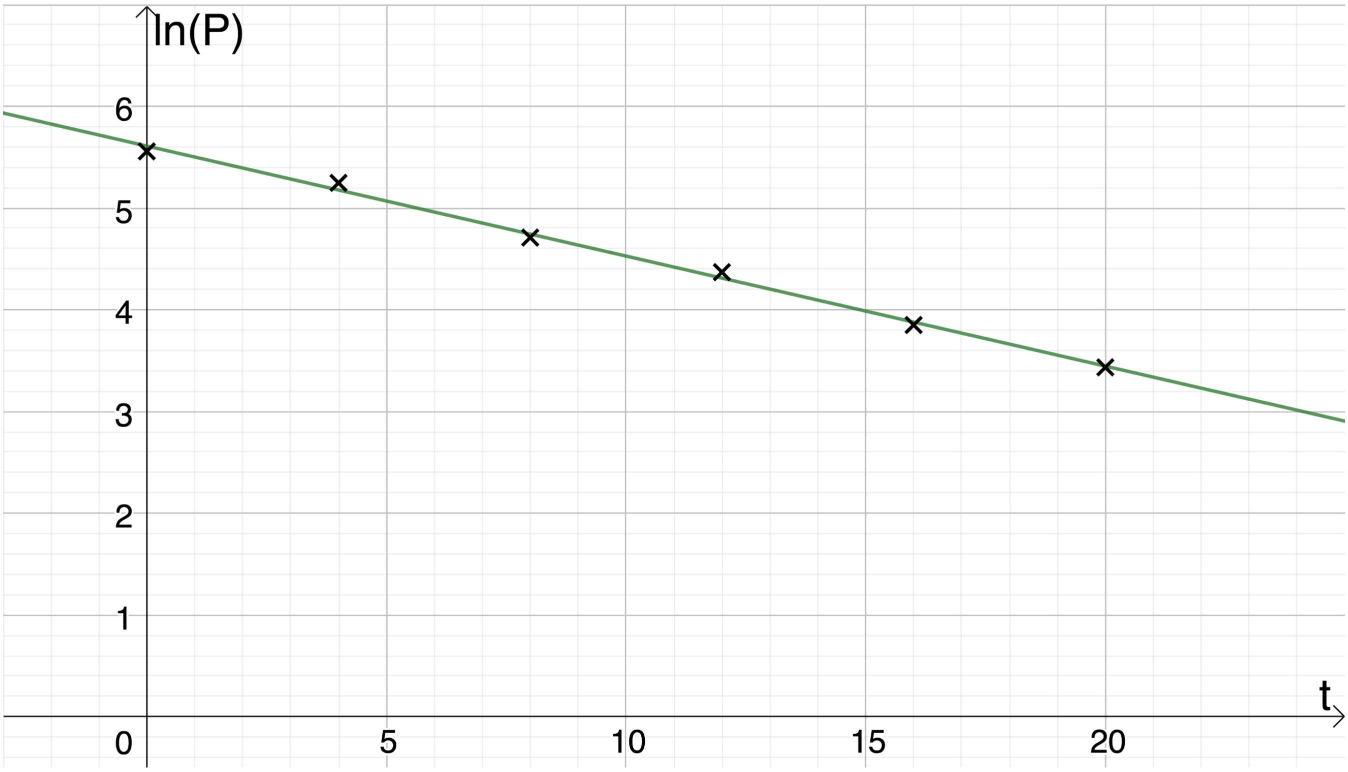
a. Show that, using this model, the graph of $\ln P$ against t is a straight line of gradient b. State the intercept of this line on the vertical axis.

$$P = ab^t \Rightarrow \ln P = \ln(ab^t)$$
$$\Rightarrow \ln P = \ln a + \ln b^t$$
$$\Rightarrow \ln P = \ln a + t \ln b$$

Intercept on the vertical axis is $\ln a$

b. Complete the table of values below and plot $\ln P$ against t, drawing by eye a line of best fit.

t	0	4	8	12	16	20
P	259	190	111	79	47	31
$\ln P$	5.55683	5.24702	4.70953	4.36945	3.85015	3.43399



c. Use your graph to find the equation for P in terms of t.

$$P = 273.037 \times 0.897^t$$

d. Use your results to estimate the year in which the population will drop to 10,000.

$$10 = 273.037 \times 0.897^t \Rightarrow t = 30.42355 \ldots$$

The population will drop to 10,000 during the year 2020.

6

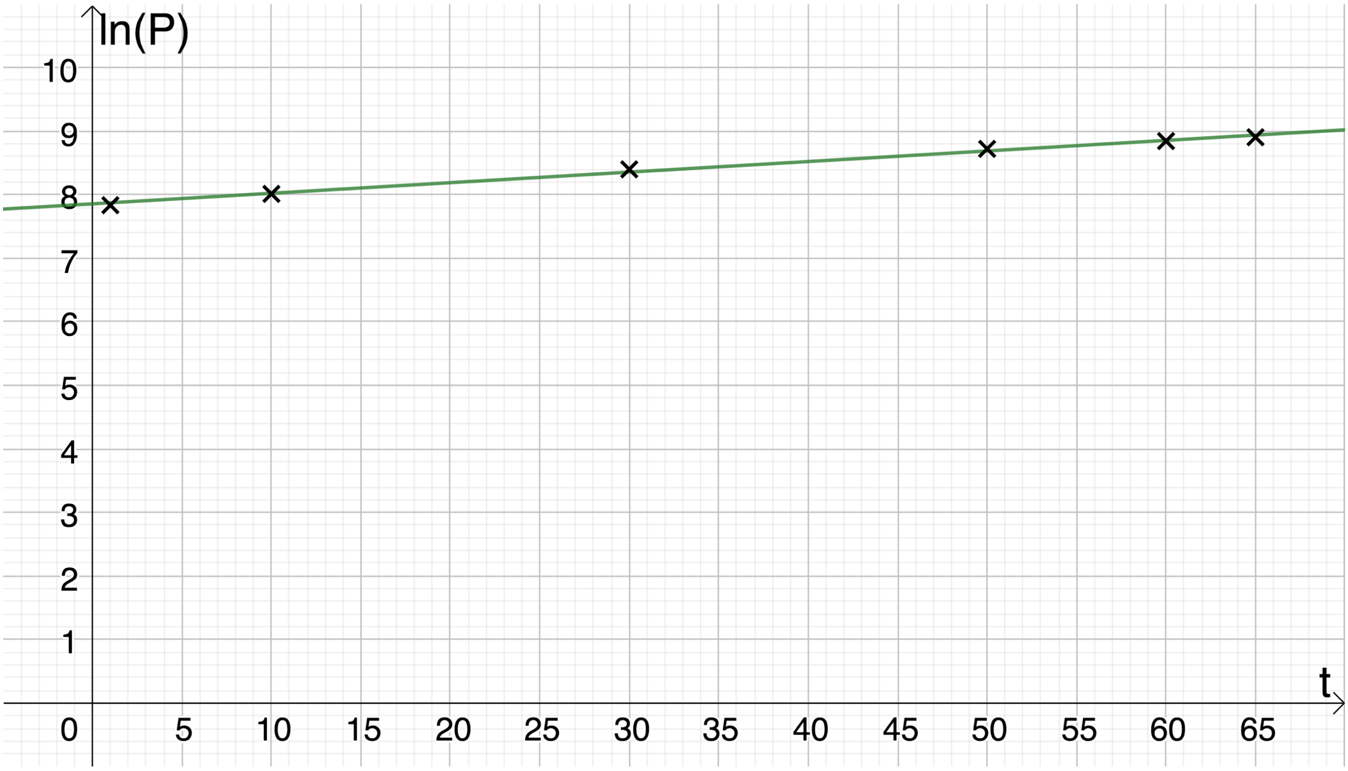
a. Show that, using this model, the graph of $\ln P$ against t is a straight line of gradient b. State the intercept of this line on the vertical axis.

$$P = ab^t \Rightarrow \ln P = \ln(ab^t)$$
$$\Rightarrow \ln P = \ln a + \ln b^t$$
$$\Rightarrow \ln P = \ln a + t \ln b$$

Intercept on the vertical axis is $\ln a$

b. Complete the table of values below and plot $\ln P$ against t, drawing by eye a line of best fit.

t	0	10	30	50	60	65
P	2525	3018	4440	6127	6930	7359
$\ln P$	7.834	8.01235	8.39841	8.72046	8.84362	8.90368



c. Use your graph to find the equation for P in terms of t.

$$P = 2580.56 \times 1.017^t$$

d. The UN projects that the population of the world will be 9735 million by 2050. Use your results to estimate the population of the world in 2050, and find the percentage difference between this and the UN's projected figure.

$$P = 2580.56 \times 1.017^{100} \approx 13925 \text{ million}$$

Percentage difference

$$= \frac{13925 - 9735}{9735} \times 100 \approx 43\%$$

7

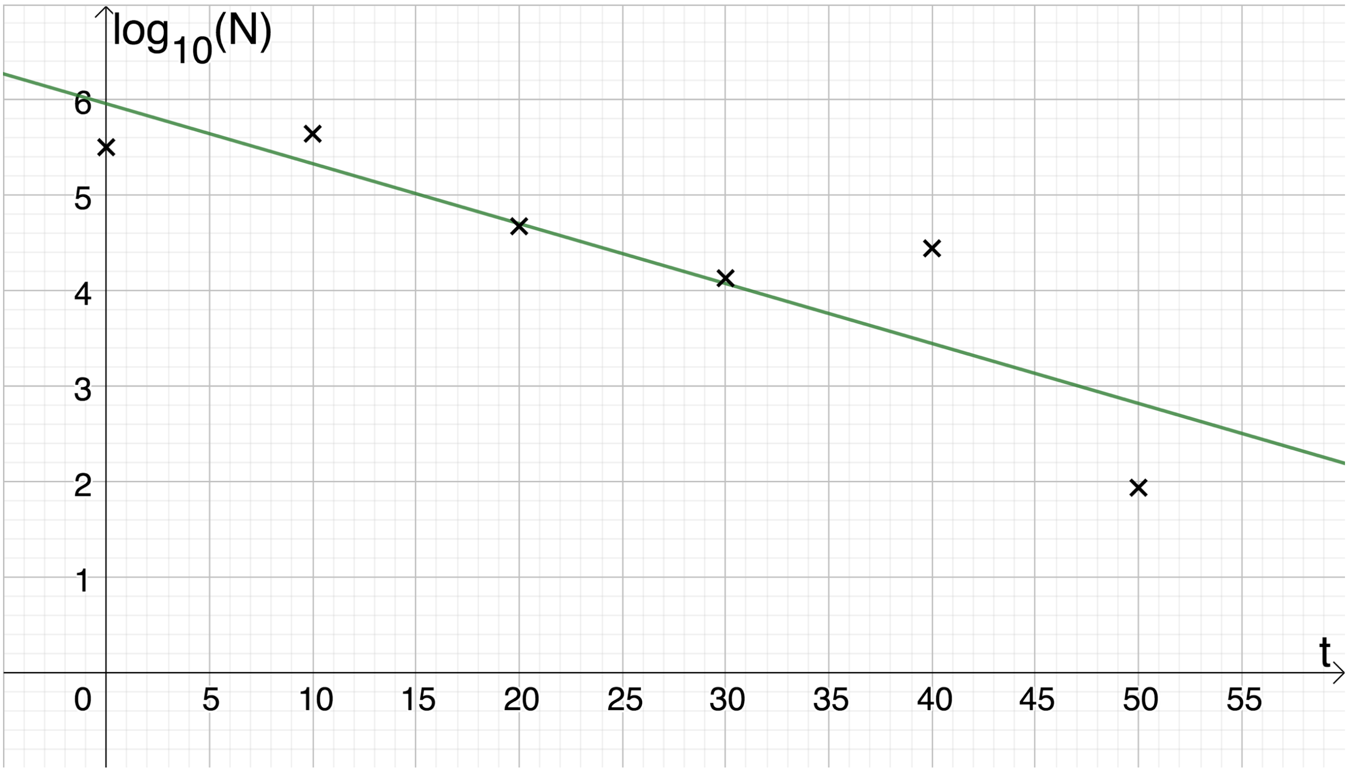
a. Show that, using this model, the graph of $\log_{10} N$ against t is a straight line of gradient b. State the intercept of this line on the vertical axis.

$$N = ab^t \Rightarrow \log_{10} N = \log_{10}(ab^t)$$
$$\Rightarrow \log_{10} N = \log_{10} a + \log_{10} b^t$$
$$\Rightarrow \log_{10} N = \log_{10} a + t\log_{10} b$$

Intercept on the vertical axis is $\log_{10} a$

b. Complete the table of values below and plot $\log_{10} N$ against t, drawing by eye a line of best fit.

t	0	10	20	30	40	50
N	319,124	441,703	47,351	13,506	27,786	86
$\log_{10} N$	12.67334	12.99839	10.76534	9.51089	10.23229	4.45435



c. Use your graph to find the equation for N in terms of t.

$$N = 912058.84 \times 0.865^t$$